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Pink-noise behaviour of biosystems

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Abstract Pink ($1/f$) noise is one of the most common behaviours of biosystems. Our present paper is devoted to clarify the origin of this interesting phenomenon. It is shown that the stationary random stochastic processes under self-similar conditions (as we have in living objects) generate pink noise independently of the kind and number of variables.

Keywords Pink noise · Biosystems · Stochastic processes

Introduction

All biosystems in vivo are energetically open and dissipative, having a dynamic equilibrium with the environment. All bioprocesses in their "living state" beside the normal entropy production are also entropy sinks, "information builders". Entropy decreases, which is one of the most peculiar physical behaviour of all living objects. This process was described by Schrodinger (1944) and called "negentropy production". In other words, the negentropy production builds energy and information into the system. The structure built-in becomes system specific by this process.

Recently, much attention has been given to the theoretical and experimental studies of the self-organization processes in various physical, chemical and biological systems (Haken 1989; Nicolis 1990). The living system is self-organizing (Walleczek 2000), which could be described by stationary, random stochastic processes (Musha and Sawada 1994). The self-organizing procedure is connected with the time fractal and a special noise, which has a power spectrum inversely proportional to the frequency ($1/f$ noise, pink noise or Flicker noise) (Schelesinger 1987; Li 1989; Voss 1989).

A new physiology, so-called "fractal physiology" (West 1990; Bassingthwaighte et al. 1994), has been developed in last few years. This new approach discusses the living structure and its dynamism in terms of spatial and temporal fractals. The typical non-linearity and chaotic behaviour characterizes most of the living processes (Cramer 1993). It is well known that there are some new diagnostic methods which are checking the noise spectrum of biosystems to control their proper function.

Power spectrum of the self-organized biomatter

Bio-objects are functionally and morphologically complex and highly organized. All biosystems are working far from the thermodynamic equilibrium, but seek to realize the lowest available energy. Its dynamic equilibrium is a stationary process, balancing the energy incorporation and the energy combustion, as well as the negentropy and entropy production. The process is self-organizing and stochastic. The take-on and -off procedure is random and the process is randomly stationary.

In order to simulate the stochastic processes, we have to base our investigation on the Fourier transform approach. Let us denote the time-dependent function (the process) by $x(t)$. Its Fourier transform is defined by (Nigam 1983):

$$X(f) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t) \mathrm{e}^{-j2\pi ft} \mathrm{d}t := \mathrm{F}\{x(t)\} \qquad (1)$$

where $j^2 = -1$.

It is easy to prove by means of the above definition that the Fourier transform of the function $x(at)$, where a is an arbitrary complex number, is:

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$$F\{x(at)\} = \frac{1}{a} X\left(\frac{f}{a}\right) \tag{2}$$

Let us define the work of the $x(t)$ process by:

$$W := \int_{-\infty}^{\infty} x^2(t)\mathrm{d}t \tag{3}$$

It follows from Parseval's formula (Nigam 1983) that this work may be evaluated by:

$$W = \int_{-\infty}^{\infty} x^2(t)\mathrm{d}t = \int_{-\infty}^{\infty} S(f)\mathrm{d}f \tag{4}$$

where the so-called spectral density function $S(f)$ is:

$$S(f) = \frac{|X(f)|^2}{2\pi} \tag{5}$$

as a function of frequency f. It can be proved that the spectral density is the even function of the frequency (Nigam 1983), i.e.:

$$S(f) = S(-f) \tag{6}$$

Note that the above-presented method cannot be generally applied in the case of stationary random processes (Nigam 1983), which is definitely our case.

By definition, a stationary random process has indefinite duration. To introduce a modified density spectrum, consider a finite segment of the random process $x(t)$ of duration $2T$, defined by:

$$x_T = \begin{cases} x(t) & -T \leq t \leq T \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

It is evident that $\lim_{T\to\infty} x_T(t) = x(t)$.

The Fourier transform of $x_T(t)$ has the form:

$$X(f,T) = \frac{1}{\sqrt{2\pi}} \int_{-T}^{T} x(t)\mathrm{e}^{-j2\pi ft}\mathrm{d}t \tag{8}$$

and:

$$F\{x_T(at)\} = \frac{1}{a} X\left(\frac{f}{a}, T\right) \tag{9}$$

In this case, Parseval's formula can be expressed as:

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\mathrm{d}t = \int_{-\infty}^{\infty} S(f)\mathrm{d}f \tag{10}$$

where:

$$S(f) = \frac{1}{2\pi} \lim_{T\to\infty} \frac{|X(f,T)|^2}{2T} \tag{11}$$

is the so-called power density spectrum in any randomly stationary case.

Power density spectrum of self-similar processes

The living procedure is basically self-similar, so it is convenient to define the self-similarity of a stochastic process. A stochastic process is said to be self-similar if the effective power of the stochastic process representation $x(t)$ equals the effective power of the representation $x(at)$ defined over time scale $[at]$, for every positive scalar $[a]$, i.e.:

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\mathrm{d}t = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(at)\mathrm{d}(at) \tag{12}$$

If we apply Parseval's formula to Eq. (7), we obtain:

$$a \int_{-\infty}^{\infty} \frac{1}{a^2} S\left(\frac{f}{a}\right)\mathrm{d}f = \int_{-\infty}^{\infty} S(f)\mathrm{d}f \tag{13}$$

where Eq. (2) was taken into account.

Also, for the power spectral density function, the functional equation may be expressed as follows:

$$S\left(\frac{f}{a}\right) = aS(f) \tag{14}$$

for every positive scalar a and every scalar f. To solve this equation, we assume that $f > 0$ and set for a the value $a = f$. Hence:

$$S(f) = \frac{S(1)}{f} \tag{15}$$

On the other hand, if $f < 0$ then $f = -|f|$; also one can write that:

$$\frac{1}{a} S\left(\frac{f}{a}\right) = \frac{1}{a} S\left(-\frac{|f|}{a}\right) = S(f) \tag{16}$$

Let us set for a the value $a = |f|$ and take into account that the power density function is even, so we obtain that:

$$S(f) = \frac{S(1)}{|f|} \tag{17}$$

This power spectrum characterizes the so-called pink ($1/f$, Flicker) noise. (In general, a stationary self-similar stochastic process follows the pink noise if its power spectral density function is proportional to $1/f$, like $S(f) \approx 1/|f|$.) So all biosystems are originally pink-noise generators, owing to their stationary stochastic processes.

Self-similarity in terms of correlation function

In accordance with the ergodic hypothesis, the autocorrelation function of a stationary random process $x(t)$ can be defined as:

$$R_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)x(t+\tau)\mathrm{d}t \tag{18}$$

which is the even function of the time shift τ, i.e.:

$$R_{xx}(\tau) = R_{xx}(-\tau) \tag{19}$$

The relation between autocorrelation function and the power density spectrum can be expressed by the Fourier transform of the autocorrelation function (Wiener-Khinchine theorem, see Nigam 1983), namely:

$$R_{xx}(f) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} R_{xx}(\tau)\mathrm{e}^{-j2\pi f\tau}\mathrm{d}\tau \tag{20}$$

or conversely:

$$R_{xx}(\tau) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} R_{xx}(f)\mathrm{e}^{j2\pi f\tau}\mathrm{d}f \tag{21}$$

From the last equation and the definition of the autocorrelation function, in the case of $\tau=0$ it follows that:

$$R_{xx}(\tau=0) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\mathrm{d}t = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} R_{xx}(f)\mathrm{d}f \tag{22}$$

Comparing this relation with Parseval's formula, we obtain:

$$R_{xx}(f) = \sqrt{2\pi}S(f) \tag{23}$$

Also the autocorrelation function can be evaluated from the power density spectrum by an inverse Fourier transform:

$$R_{xx}(\tau) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} R_{xx}(f)\mathrm{e}^{j2\pi f\tau}\mathrm{d}f = \int_{-\infty}^{\infty} S(f)\mathrm{e}^{j2\pi f\tau}\mathrm{d}f \tag{24}$$

If we apply this relation to the pink noise then it follows (Sneddon 1955) that:

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} S(f)\mathrm{e}^{j2\pi f\tau}\mathrm{d}f = \int_{-\infty}^{\infty} \frac{S(1)}{|f|}\mathrm{e}^{j2\pi f\tau}\mathrm{d}f = \frac{\sqrt{2\pi}S(1)}{|\tau|} \tag{25}$$

This remarkable result shows that the autocorrelation function of a pink noise of stationary random processes is similar both as a function of time-shift τ and frequency f. So the autocorrelation of living effects is inversely proportional to the time shift, characterizing the interdependence of the process events. This is a clear fingerprint of the self-organizing structure of living processes.

Generalized cases

Stochastic processes of one variable

In order to obtain a more general model of living objects, let us study the generalized random processes as well. By applying a little modification of the energy criteria of Eq. (7), we may assume the following relation between the effective powers of two stochastic processes $x(t)$ and $x(at)$:

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\mathrm{d}t = a^{\beta} \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(at)\mathrm{d}t \tag{26}$$

for every positive scalar a, where β is a constant.

If we apply Parseval's formula to Eq. (11), we obtain:

$$a^{\beta} \int_{0}^{\infty} \frac{1}{a^2} S\left(\frac{f}{a}\right)\mathrm{d}f = \int_{0}^{\infty} S(f)\mathrm{d}f \tag{27}$$

Also, in this case, the power spectral density function holds the same type of functional equation:

$$a^{\beta-2} S\left(\frac{f}{a}\right) = S(f) \tag{28}$$

To solve this equation, let us set for a the value $a=|f|$; hence:

$$S(f) = \begin{cases} S(1)|f|^{\beta-2}, & \text{if} f > 0 \\ S(-1)|f|^{\beta-2}, & \text{if } f < 0 \end{cases} \tag{29}$$

Since S is even function of f, the above equation implies that:

$$S(f) = S(1)|f|^{\beta-2} \tag{30}$$

In consequence, if $\beta=2$, we obtain the power spectral density of white noise, while in the case of $\beta=0$ the power spectral density of the driftless Wiener processes (Gillespie 1992) is constructed. The $\beta=1$ case trivially corresponds to pink noise. If $\beta=1$, Eq. (26) is identical with Eq. (12). The deviation of β from unity defines a non-self-similar time-scale, and so a loss of the pink-noise characteristic.

The same results can be obtained by formal rewriting of the equations for the n-dimensional vector $\overline{x(t)}$, describing a stationary random process with orthogonal coordinates $x_1(t)$, ... $x_2(t)$, ... $x_n(t)$.

Stochastic processes of many variables

The stochastic process could be given on many variables, representing independent parameters in the system which may have influence on the process. Let us take that the function $x(t_1 \ldots t \ldots t_n)$ is a representation of an n-variables stationary stochastic process. Let us assume that between the generalized effective powers of two realizations $x(t_1 \ldots t, \ldots t_n)$ and $x(a_1t_1, \ldots a_it_i, \ldots a_nt_n)$ of this stochastic process the relation may be expressed as:

$$\lim_{T_i\to\infty}\frac{1}{\prod_1^n 2T_i}\int_{-T_1}^{T_1}\ldots\int_{-T_n}^{T_N} x^2(t_1,\ldots t_i,\ldots t_n)\prod_1^n \mathrm{d}t_i$$
$$=\prod_1^n a_i^{\beta_i}\lim_{T_i\to\infty}\frac{1}{\prod_1^n 2T_i}\int_{-T_1}^{T_1}\ldots\int_{-t_N}^{T_n} x^2(a_1t_1,\ldots a_it_i,\ldots a_nt_n)\prod_1^n \mathrm{d}t_i \quad (31)$$

for every positive scalar a_i and constant β_i.

By application of Parseval's formula we obtain:

$$\prod_1^n a_i^{\beta_i}\int_{-\infty}^{\infty}\frac{1}{\prod_1^n a_i^2}S\left(\frac{f_1}{a_1},\ldots\frac{f_i}{a_i},\ldots\frac{f_n}{a_n}\right)\prod_1^n \mathrm{d}f_i$$
$$=\int_{-\infty}^{\infty}S(f,\ldots f_i,\ldots f_n)\prod_1^n \mathrm{d}f_i \quad (32)$$

from which the following functional equation may be written for the power density spectrum:

$$\prod_1^n a_i^{\beta_i-2}S\left(\frac{f_1}{a_1},\ldots\frac{f_i}{a_i},\ldots\frac{f_n}{a_n}\right)=S(f,\ldots f_i,\ldots f_n) \quad (33)$$

Here, the power density is an even function of every variable.

By applying a little modification to the above calculations, the solution of this equation will be as follows:

$$S(f,\ldots f_i,\ldots f_n)=\prod_1^n |f_i|^{\beta_i-2}S(1,\ldots 1,\ldots 1) \quad (34)$$

If $\beta_i = 1$ (for any i) then the above power density function reduces to the density function of the n-dimensional noise, which is pink noise at any variable. This result emphasizes the important internal behaviour of the self-similarity of the life represented as a stationary stochastic process.

Case of countable infinite dimensions

It is easy to see that these results may be generalized to the case of countable infinite dimensions. Now, let $\overline{x(t)}$ be an uncountable infinite dimensions vector of a stationary random process with orthogonal coordinates $x(t,\tau)$, $\tau\in(0,\infty)$. The effective power of $\overline{x(t)}$ may be defined as:

$$\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\bar{x}^2(t)\mathrm{d}t=\int_0^{\infty}\left(\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}x^2(t,\tau)\mathrm{d}t\right)\mathrm{d}\tau \quad (35)$$

Let us assume the following relation between the effective powers of two realizations $\{\bar{x}(t), \bar{x}(at)\}$:

$$\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\bar{x}^2(t)\mathrm{d}t=a^{\beta}\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\bar{x}^2(at)\mathrm{d}t \quad (36)$$

for every positive scalar a and constant β. Then Parseval's formula implies the following relation between the power spectrum densities:

$$S(f)=\int_0^{\infty}S(f,\tau)\mathrm{d}\tau=a^{\beta-2}S\left(\frac{f}{a}\right)=a^{\beta-2}\int_0^{\infty}S\left(\frac{f}{a},\tau\right)\mathrm{d}\tau \quad (37)$$

The solution for $S(f)$ is obtained by the calculation above:

$$S(f)=\int_0^{\infty}S(f,\tau)\mathrm{d}\tau=S(1)|f|^{\beta-2} \quad (38)$$

which implies the following equation:

$$\int_0^{\infty}S(f,\tau)\mathrm{d}\tau=|f|^{\beta-2}\int_0^{\infty}S(1,\tau)\mathrm{d}\tau \quad (39)$$

This equation does not have any unique solution for $S(f,\ \tau)$. The power spectrum density $S(f,\ \tau)$ has the following form:

$$S(f,\ \tau)=C\frac{1}{1+(f\tau)^2} \quad (40)$$

where C is a constant.

This is a solution in the case of $\beta = 1$. Actually, the integration of $S(f,\ \tau)$ leads to the desired result:

$$S(f)=\int_0^{\infty}S(f,\tau)\mathrm{d}\tau=\frac{\pi C}{2}\frac{1}{|f|} \quad (41)$$

from which there follows that the stationary random process $\bar{x}(t)$ is pink noise in this case as well.

Clearly, it may be enabled that the random process $\bar{x}(t)$ has complex number coordinates $x(t, \tau),\ \tau \in [0, \infty)$. In this case the effective power of $\bar{x}(t)$ must be defined by:

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \bar{x}(t)\bar{x}^*(t)\mathrm{d}t = \int_0^\infty \left(\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t,\ \tau)x^*(t,\ \tau)\mathrm{d}t \right) \mathrm{d}\tau \tag{42}$$

and the procedure is applicable in this case in the same way as above.

Conclusions

In our present work we have discussed an explanation of the pink noise generated by stationary random stochastic processes. It has been shown that the energy equation under self-similar conditions is enough to generalize pink noise in a given system described by any kind and number of variables. These conditions make it possible to understand the common pink-noise behaviour of living objects, which with self-similarity in their system and stationary random stochastic processes in their self-organizing dynamism is readily presented.

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